Magnetic Field Calculations

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I. Introduction

Current density vectors were computed for a transmembrane midwall point stimulation of a padded rat cardiac tissue sample. The sample volume contained explicit cleavage plane representations. From these current density vector fields, the magnetic flux density vector field was computed. This document contains some of these results.

II. Methods

Given a mesh of current density values known at points $\boldsymbol{x}_j$ and a mesh of points at which the magnetic flux density is to be computed, $\boldsymbol{x}_i$, the components of the vector potential field, $\boldsymbol{a}$, are calculated approximately as:

$$A_k(\boldsymbol{x}_i) \approx \frac{\mu_0}{4\pi} \sum_j \frac{J_k V_j}{|\boldsymbol{x}_i - \boldsymbol{x}_j|} \tag{1}$$

Here μ_0 is the free space permeability ($4\pi \times 10^{-10}\ H\ mm^{-1}$), J_k is the k component of the total (intra- plus extracellular) current density and V_j is the volume associated with point j.

In order to calculate the magnetic flux density vector field, $\boldsymbol{b}$, the curl of the vector potential field is required. Thus, only the gradients of $\boldsymbol{a}$ are explicitly calculated using a finite difference method.

$$\frac{\partial A_k}{\partial (x_l)_i} \approx \frac{A_k\left((x_l)_i + \Delta\right) - A_k\left((x_l)_i - \Delta\right)}{2\Delta} \tag{2}$$

$$= \frac{\mu_0}{4\pi(2\Delta)} \sum_j J_k V_j \left(\frac{1}{|(x_l)_i + \Delta - (x_l)_j|} - \frac{1}{|(x_l)_i - \Delta - (x_l)_j|} \right) \tag{3}$$

The magnetic flux density is then trivially determined.

III. Results

Transmembrane current ($4000\ \mu A\ mm^{-3}$) was injected for $2ms$ into the midwall region of a finite volume model of a sample of rat cardiac tissue. The sample contained explicit representations of cleavage planes and a full fibre, sheet and cross sheet description of intra-

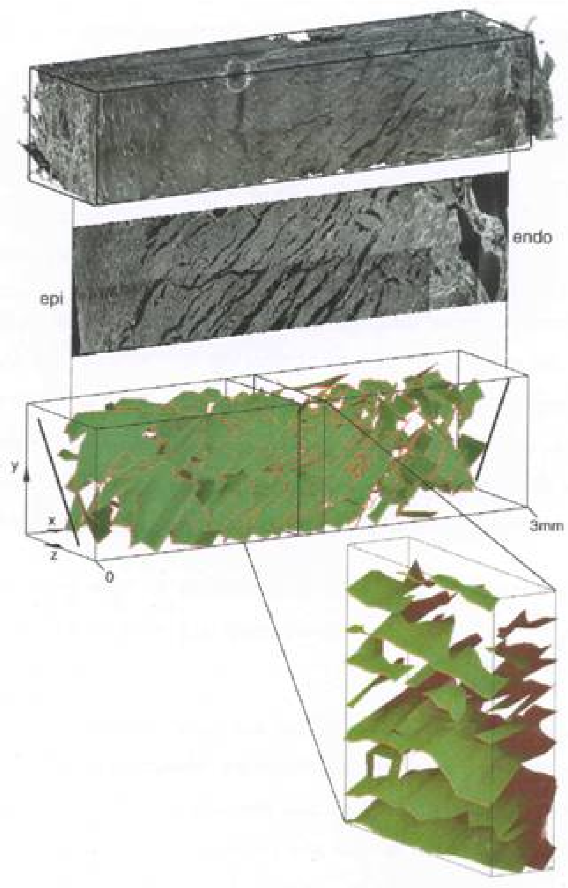
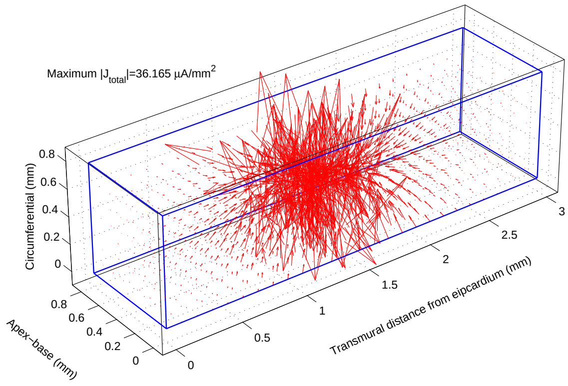


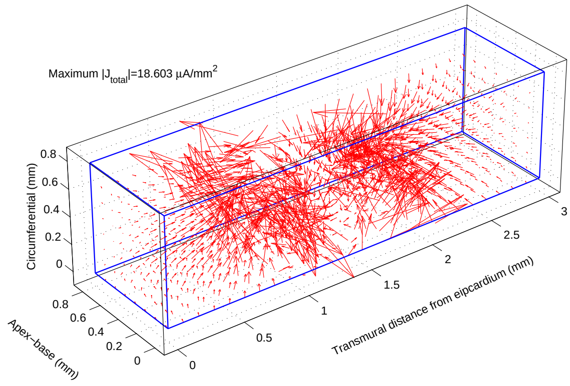
Fig. 1. Scan images of transmural rat cardiac tissue with segmented cleavage planes. The x axis is transmural, the y axis is from apex to base and the z axis is circumferential. (From Hookes et al. 2002).

and extracellular conductivities. The sample measured $3\ mm \times 0.8\ mm \times 0.8\ mm$ and was padded in the circumferential and apex-base directions by $1\ cm$ of continuous tissue padding. Current density vectors were computed and saved at $5\ ms$ intervals from $5\ ms$ to $50\ ms$. The current density vectors from the tissue sample (not including the padding volume) were used to calculate the magnetic flux density fields on an arbitrary grid.

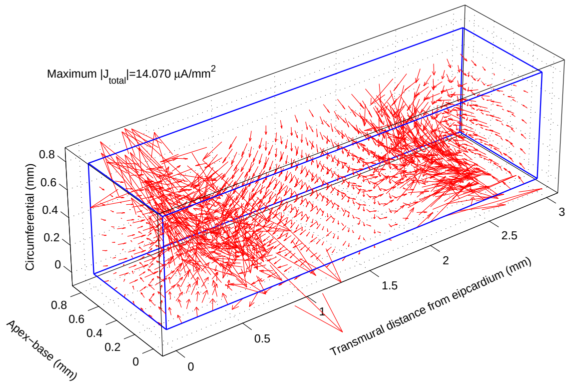
Figure 1 shows the cleavage planes and the tissue sample with fibre orientation. Figures 2 and 3 show the current density fields giving rise to the magnetic fields. Figures 4 and 5 show a selection of magnetic fields $1\ mm$ above the tissue surface.



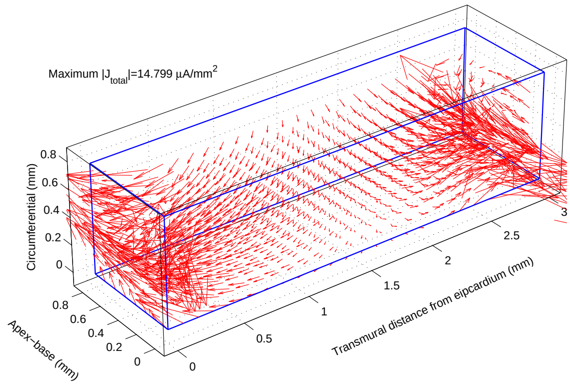
(a) 5 ms.



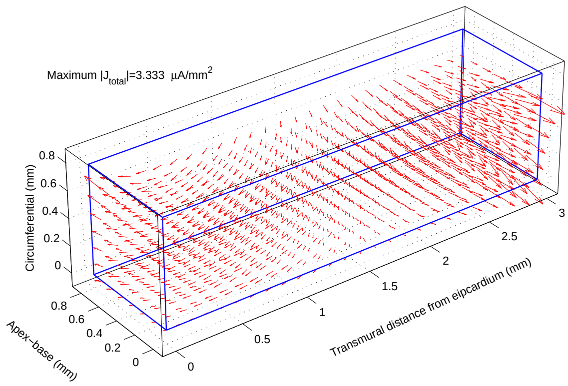
(b) 10 ms.



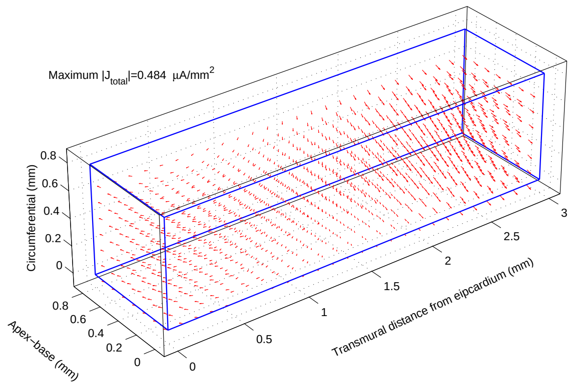
(c) 15 ms.



(d) 20 ms.

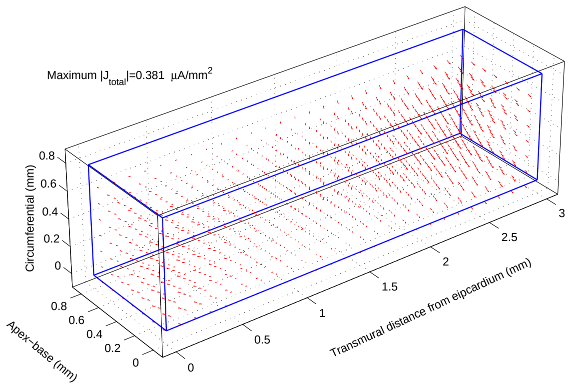


(e) 25 ms.

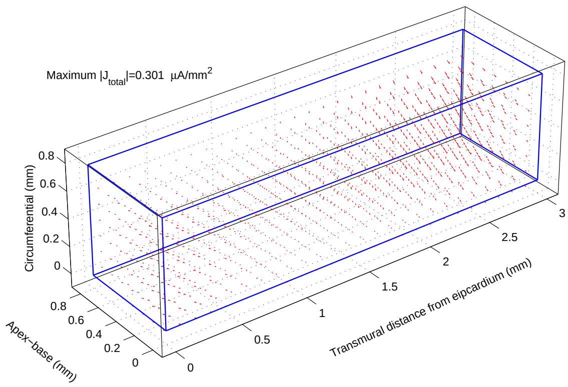


(f) 30 ms.

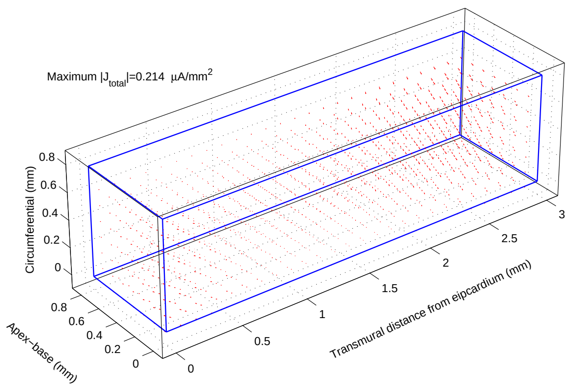
Fig. 2. Current density vectors.



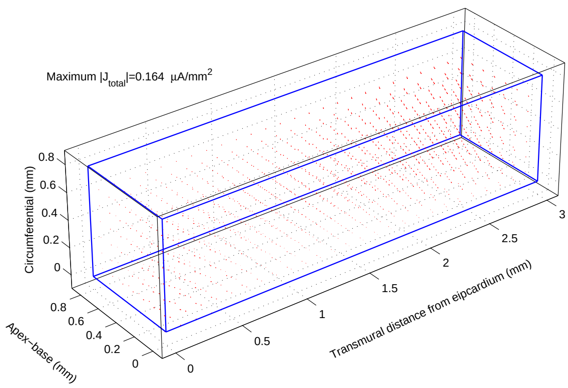
(a) 35 ms.



(b) 40 ms.

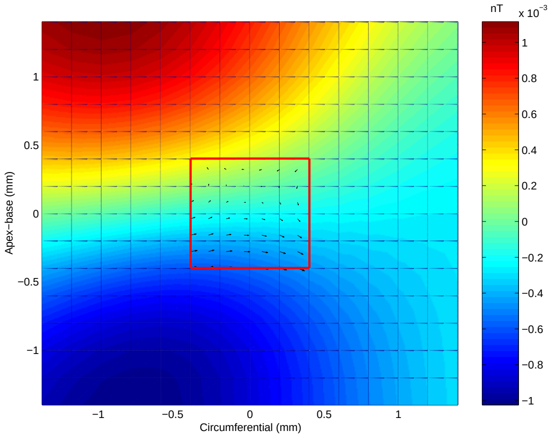


(c) 45 ms.

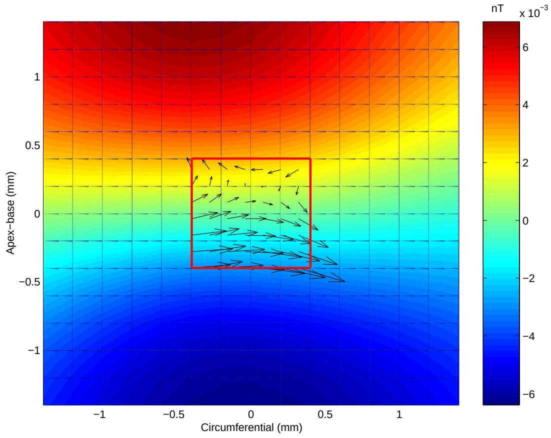


(d) 50 ms.

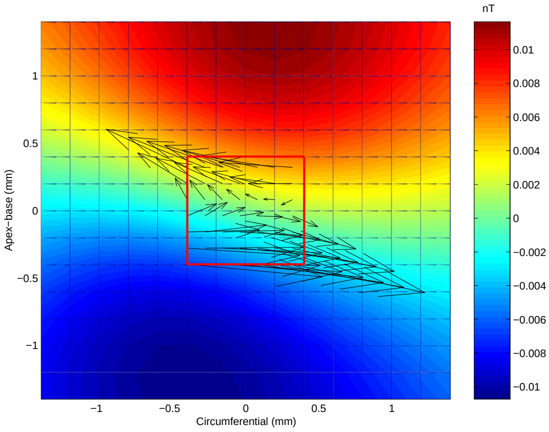
Fig. 3. Current density vectors.



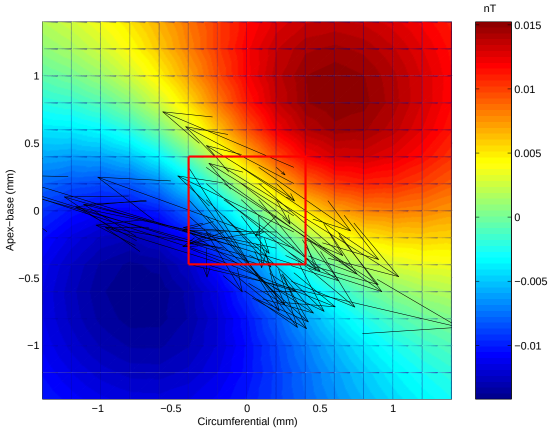
(a) 5 ms.



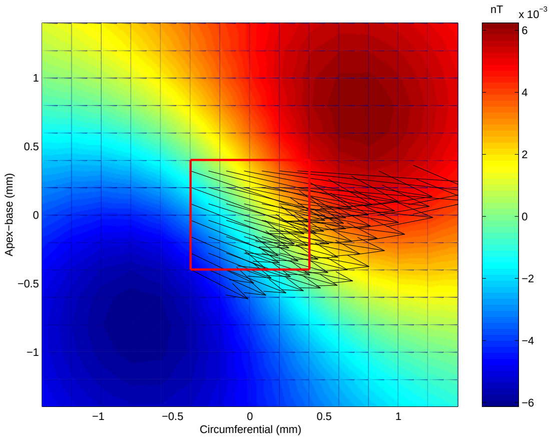
(b) 10 ms.



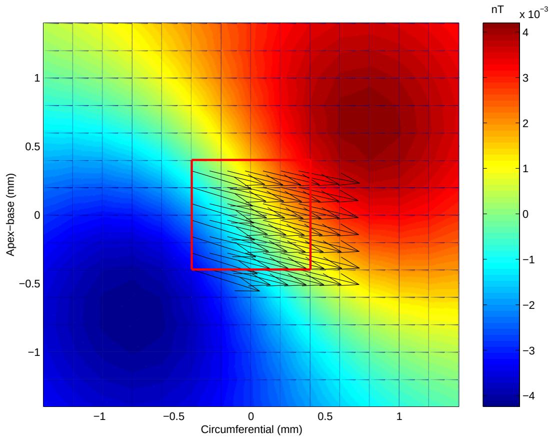
(c) 15 ms.



(d) 20 ms.

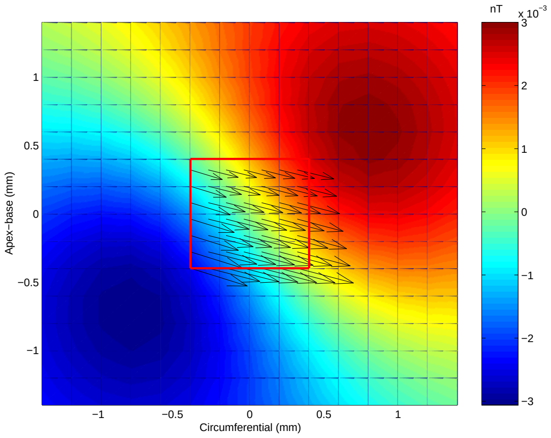


(e) 25 ms.

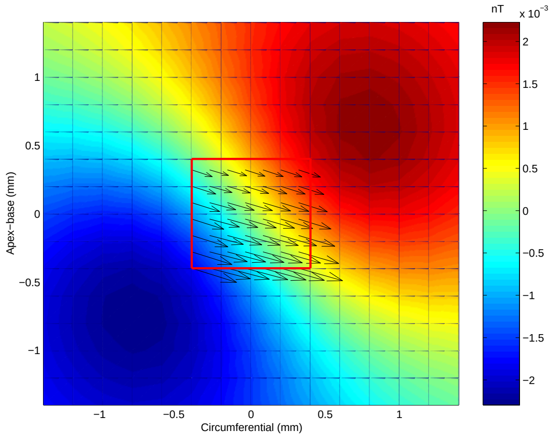


(f) 30 ms.

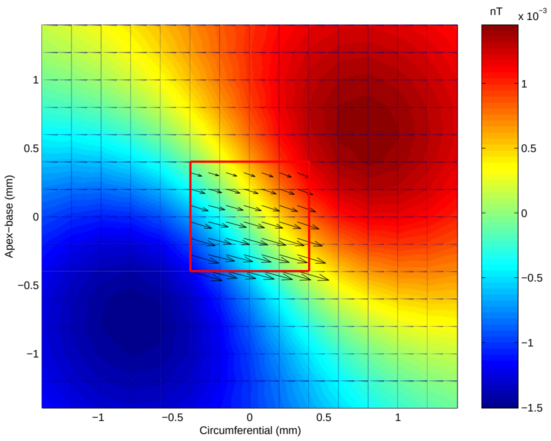
Fig. 4. Magnetic flux density 1mm from epicardial surface.



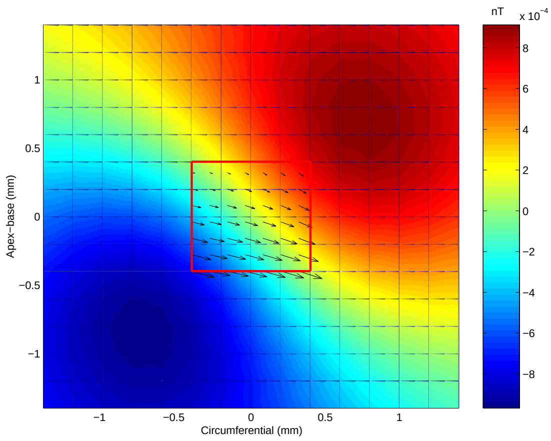
(a) 35 ms.



(b) 40 ms.



(c) 45 ms.



(d) 50 ms.

Fig. 5. Magnetic flux density 1mm from epicardial surface.